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Don't Choke

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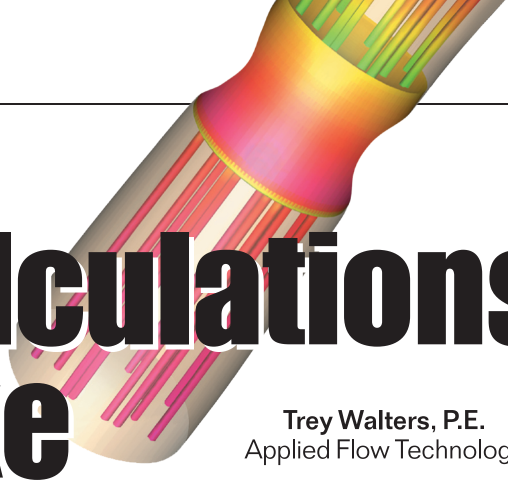

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Assuming incompressible flow simplifies the math, but introduces error. Always know how much

Flow of gases in pipe systems is commonplace in chemical-process plants. Unfortunately, the design and analysis of gas-flow systems are considerably more complicated than for liquid (incompressible) flow, due mainly to pressure-induced variations in the gas-stream density and velocity. Here, we review practical principles and present some key equations governing gas flow, and assess several assumptions and rules of thumb that engineers sometimes apply in order to simplify gas-flow analysis and calculations.

Compressible, incompressible

In a broad sense, the appropriate term for gas flow is compressible flow. In a stricter sense, however, such flow can be categorized as either incompressible or compressible, depending on the amount of pressure change the gas undergoes, as well as on other conditions.

Accurately calculating truly compressible flow in pipe systems, especially in branching networks, is a formidable task. Accordingly, engineers often apply rules of thumb to a given design situation involving gas flow, to decide whether the use of (simpler) incompressible-flow calculations can be justified. Such rules of thumb are helpful, but they can lead one astray when used without a full understanding of the underlying assumptions.

Sometimes, the case is clear-cut. For instance, if the engineer is designing a near-atmospheric-pressure ventilation system, with pressure drops measured in inches of water, incompressible-flow methods are perfectly suitable. Conversely, for design or specification of a pressure-relief system that is certain to experience high velocities, compressible-flow methods will clearly be required. In practice, many gas systems fall between these extremes, and it is difficult to assess the error that will result from using incompressible methods.

A major purpose of this article is to offer guidelines for assessing the importance of compressibility effects in a given case. First, however, we set out relevant equations, and discuss some key aspects of gas-flow behavior.[1]

The underlying equations

Incompressible flow: An apt starting point for discussing gas flow is an equation more usually applied to liquids, the Darcy-Weisbach equation (see Nomenclature box, next page):

$$\Delta P = -\left[f\frac{L}{D}\left(\frac{1}{2}\rho V^2\right)+\rho g dz\right] \qquad (1)$$

where f is the Moody friction factor, generally a function of Reynolds number and pipe roughness. This equation assumes that the density, ρ, is constant. The density of a liquid is a very weak function of pressure (hence the substance is virtually incompressible), and density changes due to pressure are ignored in practice. The density varies more significantly with tem-perature. In systems involving heat transfer, the density can be based on the arithmetic average, or, better, the log mean temperature. When the appropriate density is used, Equation (1) can be used on a large majority of liquid pipe-flow systems, and for gas flow when compressibility can be ignored.

1. The quantitative compressible- and incompressible-flow results in this article were obtained using, respectively, AFT Arrow and AFT Fathom. Both are commercially available software for pipe system modeling. A simplified but highly useful utility program, Compressible Flow Estimator (CFE), was developed specifically for this article, and was used in several cases.

Compressible flow: Equation (1) is not strictly applicable to compressible flow because, as already noted, the density and velocity change along the pipe. Sometimes, engineers apply Equation (1) to gas flow by taking the average density and velocity. But, because the variation of each of these parameters along a pipe is nonlinear, the arithmetic averages will be incorrect. The difficult question — How seriously incorrect? — is discussed in detail later in this article.

Individual length of pipe: More strictly applicable than Equation 1 to gas flow in a pipe are Equations (2)–(6) [*1–3*], developed from fundamental fluid-flow principles and generalized from perfect gas equations [*4*] to apply to real gases:

Mass:

$$\frac{d\rho}{\rho}+\frac{dV}{V}=0 \qquad (2)$$

Momentum:

$$dP+\frac{1}{2}\rho V^2\frac{f}{D}dx+\rho V dV+\rho g dz=0 \qquad (3)$$

Energy:

$$\dot{m}d\left(h+\frac{1}{2}V^2+gz\right)=q \qquad (4)$$

Equation of State:

$$P = Z\rho RT \qquad (5)$$

Mach number:

$$M=\frac{V}{\sqrt{\gamma ZRT}} \qquad (6)$$

Several things should be noted about Equations (2)–(6):

• They assume that the pipe diameter is constant

• They are applicable not only to individual gases but also to mixtures, so long as appropriate mixture properties are used

• Equation (1) is a special case of the momentum equation, Equation (3). If the third term on the left-hand side of the latter (commonly called the ac-

NOMENCLATURE

- **A** cross-sectional flow area of a pipe
- **D** diameter of a pipe
- **e** pipe wall roughness
- **f** friction factor
- F_f, g, γ, T_0 parameters in Equation (14)
- $\bar{F}$ arithmetical average of F over computing section
- **g** acceleration (usually gravitational)
- **h** enthalpy, static
- h_0 enthalpy, stagnation
- **L** length of a pipe
- **M** Mach number
- $\dot{m}$ mass flowrate
- **P** pressure
- P_0 pressure, stagnation
- **R** gas constant
- **s** entropy
- **T** temperature, static
- T_0 temperature, stagnation
- **V** velocity
- **x** length
- **z** elevation
- **Z** compressibility factor
- γ specific heat ratio
- θ angle from horizontal
- ρ density

SUBSCRIPTS

- **1** Location 1 in pipe
- **2** Location 2 in pipe
- **i** junction at which solution is sought
- **j** junctions with pipes connecting to junction i

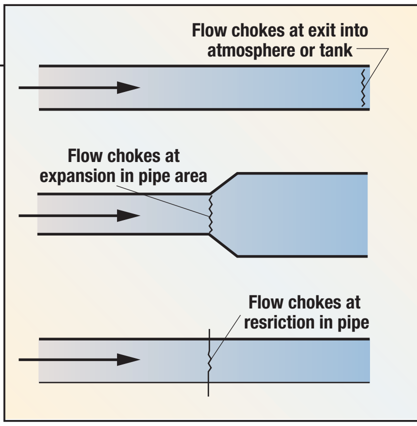


FIGURE 1. Any of these piping configurations can result in sonic choking

celeration term) is neglected, the two equations become identical

- Equation (4), the energy equation, includes the conventional thermodynamic enthalpy plus a velocity term that represents changes in kinetic energy. The sum of these two terms is known as the stagnation enthalpy (see discussion of stagnation properties, below). The thermodynamic enthalpy is referred to as the static enthalpy (even if it pertains to a moving fluid). Similarly, temperature in a non-stagnation context is referred to as static temperature
- Equation (5), as shown, includes a compressibility factor to correct the ideal gas equation for real-gas behavior. In general, however, the real-gas properties can instead be obtained from a thermophysical property database

***Piping networks**:* In situations involving a gas-pipe network, Equations (2)–(6) are applied to each individual pipe, and boundary conditions between the pipes are matched so that mass and energy are balanced. The following equations describe this balance at any branch connection:

Mass balance:

$$\sum_{j=1}^{n} \dot{m}_{ij} = 0 \qquad \textbf{(7)}$$

Energy balance:

$$\sum_{j=1}^{n} \dot{m}_{ij}\left(h_{ij} + \frac{1}{2}V_{ij}^2\right) = 0 \qquad \textbf{(8)}$$

In Equation (8) (in essence, a statement of the First Law of Thermodynamics), energy is balanced by summing (for each pipe at the branch connection) the mass flowrate multiplied by the stagnation enthalpy. Elevation effects drop out, because all elevations at the connection are the same.

If gas streams of different composition mix at a branch connection, a balance equation will also be needed for each individual species present. Additional discussion of species balance can be found in Reference [*3*]. Use of these network-calculation principles is discussed in more detail later.

Besides the use of the basic equations set out above, gas-flow designs and calculations also frequently involve two concepts that are usually of lesser or no importance with incompressible flow: stagnation conditions, and sonic choking.

Stagnation conditions

At any point in a pipe, a flowing gas has a particular temperature, pressure and enthalpy. If the velocity of the gas at that point were instantaneously brought to zero, those three properties would take on new values, known as their stagnation values and indicated in the equations of this article by the subscript 0.

Three important stagnation conditions can be calculated, for real as well as ideal gases, from the velocity and the specific heat ratio (ratio of specific heat at constant pressure to that at constant volume) by Equations (9 a, b and c). As is frequently the case in gas flow, the velocity is expressed in terms of the Mach number:

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2}M^2 \qquad \textbf{(9a)}$$

$$\frac{P_0}{P} = \left[1 + \frac{\gamma - 1}{2}M^2\right]^{\frac{\gamma}{\gamma - 1}} \qquad \textbf{(9b)}$$

$$h_0 = h + \frac{V^2}{2} \qquad \textbf{(9c)}$$

Sonic choking

In almost all instances of gas flow in pipes, the gas accelerates along the length of the pipe. This behavior can be understood from Equations (2), (3) and (5). In Equation (3), the pressure falls off, due to friction. As the pressure drops, the gas density will also drop (Equation [5]). According to Equation (2), the dropping density must be balanced by an increase in velocity to maintain mass balance.

It is not surprising, then, that gas flow in pipelines commonly takes place at velocities far greater than those for liquid flow — indeed, gases often approach sonic velocity, the local speed of sound. A typical sonic velocity for air is 1,000 ft/s (305 m/s).

When a flowing gas at some location in the pipeline experiences a local velocity equal to the sonic velocity of the gas at that temperature, sonic choking occurs and a shock wave forms. Such choking can occur in various pipe configurations (Figure 1).

The first case, which can be called endpoint choking, occurs at the end of a pipe as it exits into a large vessel or the atmosphere. In this situation, the gas pressure cannot drop to match that at the discharge without the gas accelerating to sonic velocity. A shock wave forms at the end of the pipe, resulting in a pressure discontinuity.

The second case, which might be called expansion choking, crops up when the cross-section area of the pipe is increased rapidly; for example, if the system expands from a 2-in. pipe to one of 3-in. pipe. This can also happen when a pipe enters a flow splitter such that the sum of the pipe areas on the splitting side exceeds the area of the supply pipe. A shock wave forms at the end of the supply pipe, and a pressure discontinuity is established.

(*Continues on next page*)

The third case, which may be called restriction choking, occurs when the gas flows through a restriction in the pipe, such as an orifice or valve. In such a case, the flow area of the gas is reduced, causing a local increase in velocity, which may reach the sonic velocity. A shock wave forms at the restriction, with a pressure discontinuity similar to the first two cases.

Figure 2 shows stagnation-pressure and Mach-number profiles for expansion choking and restriction choking; both involve supply air at 100 psia and 1,000°R discharging to 30 psia. Endpoint-choking behavior appears in Figure 7, discussed later.

For a given process situation, the choked flowrate can be determined from Equation (10a), by inserting a Mach number of 1 into Equation (10b):

$$\dot{m} = A\frac{P_0}{\sqrt{T_0}}f(M) \qquad \textbf{(10a)}$$

where:

$$f(M) = \sqrt{\frac{\gamma}{ZR}}M\left(1+\frac{\gamma-1}{2}M^2\right)^{-(\gamma+1)/[2(\gamma-1)]} \qquad \textbf{(10b)}$$

These equations can be derived from the continuity equation [*4*, p.97].

In practice, it is difficult to apply these equations to choked conditions, because the local conditions, P_0 and T_0, are not known at the point of choking. For instance, to apply the equations to endpoint choking, one must calculate the stagnation pressure and temperature at the end of the pipe, upstream of the shock wave — but these two variables depend on the flowrate, which is not yet known.

The only way to solve such a problem accurately is by trial and error: first, assume a flowrate and march down the pipe; if M reaches 1 before the end of the pipe, repeat the procedure with a lower assumed flowrate; repeat until M reaches 1 right at the pipe endpoint. Obviously, this calculation sequence is not practical without a computer.

From the standpoint of pipe design or system operation, sonic choking sets a limit on the maximum possible flowrate for a given set of supply conditions. In particular, lowering the discharge pressure does not raise the flowrate. Figure 3 illustrates this for

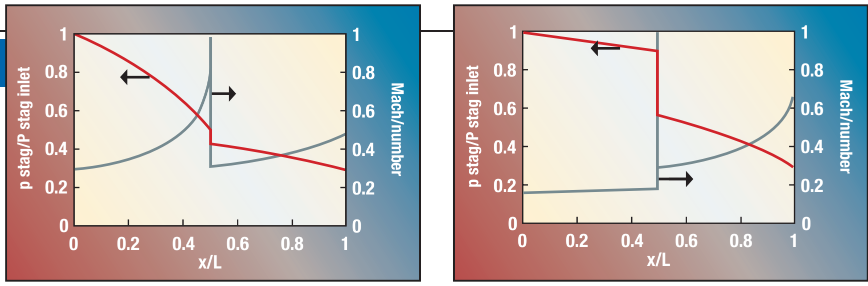


FIGURE 2. These stagnation-pressure and Mach-number profiles are for (left) expansion choking, involving a 2-in. pipe expanding to 3 in., and (right) restriction choking at a 0.6-area-ratio orifice in a 2-in. pipe

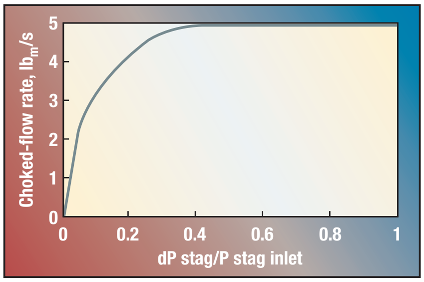


FIGURE 3. In this adiabatic flow of 100-psia, 70°F air, sonic choking occurs at 63.6-psia or lower discharge pressure

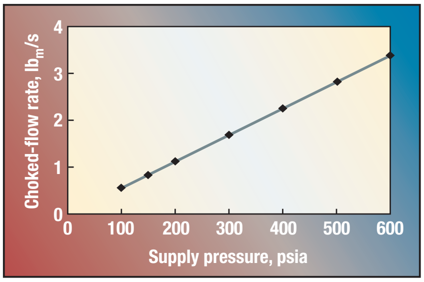


FIGURE 4. Increasing the supply pressure raises the choked flowrate (shown here for an adiabatic flow of steam)

a 2-in. pipe carrying air that is supplied at 100 psia. Despite containing no physical restrictions, this system experiences endpoint choking at any discharge pressure below 63.6 psia.

Some engineers misapply the concept of sonic choking and conclude that the sonic flowrate is the maximum possible through a given system for all conditions. In fact, however, the flowrate can be increased by raising the supply pressure. Indeed, the increased choked-flowrate presumably increases linearly with increased supply pressure (Figure 4).

The pressure drop across the shock wave in choked flow cannot be calculated directly.[2] The only recourse is to use the choked flowrate as a new boundary condition on the pipe downstream of the shock wave (assuming that one is not dealing with endpoint choking) and to apply Equations (2) – (6) in the remaining pipes. The shock-wave process is not truly isenthalpic, but (in accordance with Equation [4]) instead entails constant stagnation enthalpy.

Be aware that a given pipe can choke at more than one location along its length. This occurs when the choked flowrate set by the upstream choke point is applied to the pipes beyond the upstream shock wave, and the gas at this flowrate cannot reach the end of the pipe without experiencing another shock wave. In fact, there is no limit to the number of choke points in a pipe, other than the number of possible geometric configurations that permit shock waves. The three mechanisms that cause choking can all occur in the same pipeline, in any combination. References [*2*] and [*3*] discuss calculation procedures for multiple-choking systems.

Single-pipe adiabatic flow

Before presenting compressible-flow equations that are generally applicable (Equations [13] and [14]), we consider two special cases: adiabatic and isothermal flow. Both are important in their own right. What's more, analysis of the two (see below) leads to the guidelines that can help the engineer decide whether compressibility (with its far more-complex calculations) must be taken into account in a given process situation.

The thermodynamic process a gas undergoes in constant-diameter adiabatic flow can be viewed in terms of entropy and static enthalpy. This process traces out a curve called the Fanno line[3] (Figure 5). The Fanno line neglects elevation changes, a safe assumption in most gas systems.

According to the Second Law of Thermodynamics, the entropy increases as the gas flows through the pipe. Thus, depending on the initial state of the gas (either subsonic or supersonic), the

2. "Normal shock tables" (perhaps more familiar to aeronautical engineers than to chemical engineers) apply only to supersonic flows, and are of no use for sonic or subsonic pipe flow.

3. Some authors show the Fanno line as a plot of temperature (rather than enthalpy) vs. entropy.

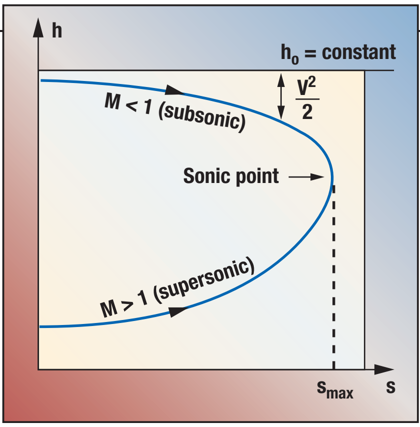
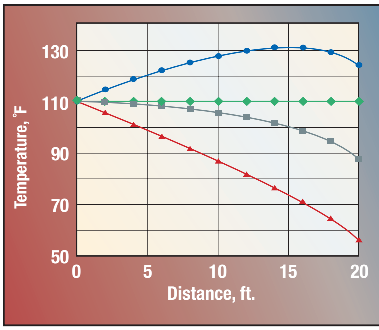


FIGURE 5. Fanno lines, such as the one presented here, show enthalpy vs. entropy for adiabatic flow in a pipe



Case	Mass flow (lb/s)
Cooled	0.9197
Adiabatic	0.8994
Isothermal	0.8903
Heated	0.8776

FIGURE 6. Adiabatic and isothermal flow do not bracket gas flowrates. In four situations shown here, 100-psia, 111°F air is fed into a 1-in. pipe 20 ft long. Outlet pressure is 60 psia. Cooled flow has 30°F ambient temperature; heated flow, 220°F. The heat-transfer coefficients are 100 Btu/(h)(ft²)(°F)

process will follow either the upper or lower portion of the curve. Very few process situations entail supersonic flow in pipes, so we will focus on the subsonic (i.e., upper) portion.

The stagnation enthalpy, h_0, is constant because the system is adiabatic. However, the gas is accelerating, which causes the static enthalpy to decrease, in accordance with Equation (4). If the proper conditions exist, the gas will continue to accelerate up to the point at which its velocity equals the sonic velocity, where sonic choking begins.

As Figure 5 shows, the enthalpy approaches the sonic point asymptotically. Accordingly, the thermodynamic properties experience intensely rapid change at the end of a sonically choked pipe. Examples of such change arise later in this article.

The gas static temperature usually decreases as it travels along the pipe, due to the decreasing pressure. Under certain conditions, however, the reverse is true. The governing parameter in this regard is the Joule-Thompson coefficient [*5*, *8*]. The points made in this article are (unless otherwise noted) applicable for either the cooling or heating case if the appropriate words are substituted, but we assume the cooling case for the sake of discussion. For more on Fanno flow see References [*4*, *6*, *7*].

From Equations (2)– (6), the following equation can be derived for adiabatic flow of a perfect gas [*4*, p. 209]:

$$\int_0^L \frac{f}{D}dx = \int_{M_1}^{M_2} \frac{1-M^2}{\gamma M^4\left(1+\frac{\gamma-1}{2}M^2\right)}dM^2 \qquad \textbf{(11a)}$$

Integrating from 0 to L along the length of the pipe gives:

$$\frac{fL}{D} = \frac{1}{\gamma}\left(\frac{1}{M_1^2}-\frac{1}{M_2^2}\right) + \frac{\gamma+1}{2\gamma}\ln\left(\frac{M_1^2}{M_2^2}\right)\left(\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}\right) \qquad \textbf{(11b)}$$

Single-pipe isothermal flow

In the second special case, isothermal flow, the static temperature of the gas remains constant. As already noted, the tendency is for gas to cool as it flows along a pipe. For the temperature to remain constant, an inflow of heat is required.

When temperature is constant, Equations (2)–(6) become somewhat simpler. In Equation (5), for instance, density becomes directly proportional to pressure, and a perfect-gas analytical solution can be obtained:

$$\int_0^{L_T} \frac{f}{D}dx = \int_{M_1}^{M_2} \frac{1-\gamma M^2}{\gamma M^4}dM^2 \qquad \textbf{(12a)}$$

where the T subscript on L emphasizes that the system is isothermal.

Integrating from 0 to L gives:

$$\frac{fL_T}{D} = \frac{1-\left(\frac{M_1^2}{M_2^2}\right)}{\gamma M_1^2} - \ln\left(\frac{M_2^2}{M_1^2}\right) \qquad \textbf{(12b)}$$

To truly maintain isothermal flow up to the sonic point would require an infinite amount of heat addition. This leads to the strange but mathematically correct conclusion that for isothermal flow, sonic choking occurs at a Mach number less than 1. Practically speaking, it is not feasible to keep a gas flow fully isothermal at high velocities. For a more-complete discussion of isothermal flow in pipes, see Reference [*4*], pp. 265–269.

One occasionally finds a misconception among engineers designing gas systems: that adiabatic and isothermal flow bracket all possible flowrates. However, this is not true. First, consider the adiabatic case, where no heat is added but the gas cools. If heat is removed, the cooling will exceed that in adiabatic flow. Next consider isothermal flow, where the addition of heat keeps the gas static temperature constant. If more heat is added than required to maintain isothermal flow, the static temperature will increase.

In summary, the heat-transfer environment plays a critical role in determining whether the gas flow is closer to adiabatic or isothermal. It is also the mechanism that can cause the gas flow to exceed the limits of the two special cases. Figure 6 demonstrates the different situations.

General single-pipe equations

For the general (neither adiabatic nor isothermal) case, in situations when the compressibility of the gas cannot be ignored, Equations (2)–(6) can be combined and, through calculus and algebra [*3*, *4*], represented in differential form by Equations (13) and (14). Equation (13a) [*1-3*] is based on a fixed-length step between Locations 1 and 2 along the pipe. The terms involving | and Z account for the real-gas effects:

$$\frac{dP_0}{P_0} = -\frac{\gamma M^2}{2}\left(\frac{dT_0}{T_0}+\frac{fdx}{D}+\frac{dZ}{Z}+\frac{d\gamma}{\gamma}\right) - \frac{g\sin\theta dx}{ZRT_0}\left(1+\frac{\gamma-1}{2}M^2\right) \qquad \textbf{(13a)}$$

Integration yields:

$$P_{0,2} = P_{0,1}e^{-C} \qquad \textbf{(13b)}$$

where:

$$C = \frac{\gamma M^2}{2}\left(\ln\frac{T_{0,2}}{T_{0,1}}+\ln\frac{Z_2}{Z_1}+\ln\frac{\gamma_2}{\gamma_1}+\frac{f(x_2-x_1)}{D}\right) + \frac{g\sin\theta(x_2-x_1)}{RZT_0}\left(1+\frac{\gamma-1}{2}M^2\right) \qquad \textbf{(13c)}$$

Conditions at Location 1 are known;

the goal is to find those at Location 2 that satisfy the equations. There are multiple unknowns at Location 2, and much iteration is required.

In addition, some expression for the heat-transfer process is required in order to apply the energy equation, Equation (4). In a convective application, this will usually require a convection coefficient. For more details, see Reference [*3*].

Another formulation of these equations is better suited for systems that incur either endpoint or expansion sonic choking. This method takes solution steps over equal Mach-number increments rather than length increments [1-3]:

$$\frac{dM^2}{M^2} - F_{T_0}\left(\frac{dT_0}{T_0} + \frac{dZ}{Z}\right) - F_\gamma \frac{d\gamma}{\gamma} - F_f \frac{f dx}{D} - F_g \frac{g \sin\theta dx}{ZRT_0} = 0 \quad \textbf{(14a)}$$

where:

$$F_{T_0} = \frac{\left(1+\gamma M^2\right)\left(1+\frac{\gamma-1}{2}M^2\right)}{1-M^2} \quad \textbf{(14b)}$$

$$F_\gamma = \frac{\left(1-\gamma M^2\right)\left(1+\frac{\gamma-1}{2}M^2\right)}{1-M^2} \quad \textbf{(14c)}$$

$$F_f = \frac{\gamma M^2\left(1+\frac{\gamma-1}{2}M^2\right)}{1-M^2} \quad \textbf{(14d)}$$

$$F_g = 1+\frac{\gamma-1}{2}M^2 \quad \textbf{(14e)}$$

Integration yields:

$$x_2 = x_1 + \frac{\ln\frac{M_2^2}{M_1^2} - \overline{F}_{T_0}\left(\ln\frac{T_{0,2}}{T_{0,1}} + \ln\frac{Z_2}{Z_1}\right) - \overline{F}_\gamma\left(\ln\frac{\gamma_2}{\gamma_1}\right)}{\overline{F}_f \frac{f}{D} + \overline{F}_g \frac{g \sin\theta}{RZT_0}} \quad \textbf{(14f)}$$

An increase in Mach number from M_1 to M_2 can be arbitrarily specified (say, by increments of 0.01); then, one computes the distance from x_1 to x_2 that is required to obtain this change in Mach number. Again, extensive iteration is required because there are multiple unknowns at Location 2. This method lets the engineer follow the rapidly changing conditions at the end of the pipe during choking (see Figure 7).

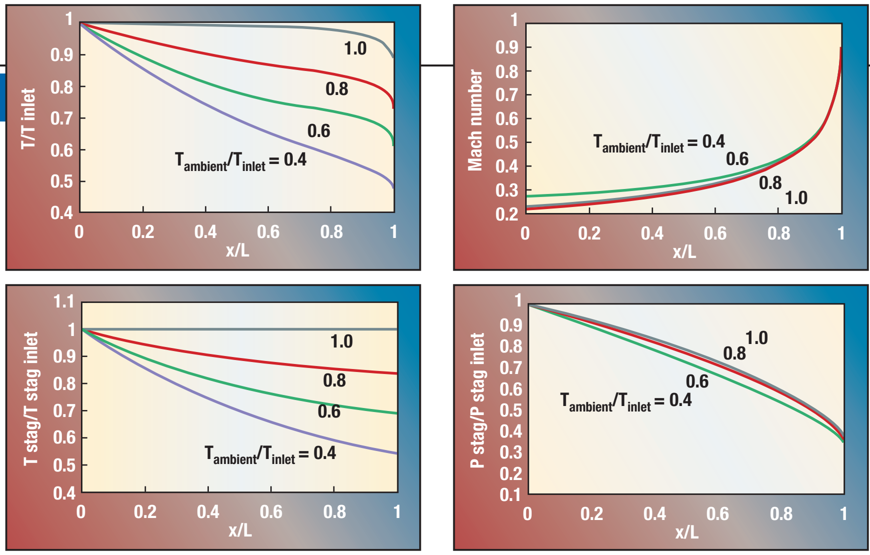


FIGURE7. These typical, dimensionless property variations were taken with respect to air supplied at 100 psia and 1,000±R into a 2-in. pipe 100 ft long, under conditions providing a sonically choked discharge. The curves relate to four ambient-temperature conditions. Similar curves could be drawn showing other parameters, such as density, enthalpy and static pressure, as functions of the distance along the pipe

Simplification error: How big?

As already noted, a key question arises: How much error is introduced if the engineer sidesteps the calculational complications of equations such as Equations (13) and (14) by instead making the incompressible-flow assumption?

Adiabatic flow: In the fully adiabatic-flow case (that is, assuming a perfectly insulated pipe), Figure 8 provides typical answers to that question, with respect to three specific cases. They involve, respectively, the flow of three widely used fluids: air, steam and methane (the last-having properties similar to those of natural gas).

The results in Figure 8 were developed by building models for both compressible and incompressible flow. The latter models used the arithmetic average fluid density, and assumed that the viscosity was constant. The inlet stagnation conditions for the three streams were as follows:

Air: 100 psia, 70° F
Steam: 500 psia, 600° F
Methane: 500 psia, 100° F

All pipes were standard steel, with a roughness of 0.00015 ft.

With respect to each of the three gases, we compared the calculated flowrates for the two cases. The difference between the two is the error that results from using the incompressible assumption. The error is plotted in Figure 8 for 1-in. pipe of three different lengths.

The clustering of the air, steam and methane results confirms that the pipe pressure-drop ratio and the ratio of length to diameter are appropriate parameters to use for generalization when focusing on a specific pipe diameter.

For the conditions modeled, air followed the ideal gas law closely. However, the steam and methane conditions did not follow the ideal gas law, with compressibility factors (corrections for non-ideality) ranging from 0.92 to 0.97. From these data, it appears that the generalizations implied by Figure 8 can be applied to non-ideal gases.

To extend the generalization, the preceding calculations were repeated for air flowing in pipes with diameters of 3, 6, 12 and 24 in., increasing the pipe length each time to maintain the *L*/*D* ratios of 50, 200 and 1,000. Results (not shown) indicate that the error is always larger than for the 1-in.dia pipe with the same *L*/*D*. For 24-in. pipe, the error is larger by over a factor of two.

Why does the incompressible-flow-assumption error increase as the pipe diameter increases? The reason relates to the pipe-roughness data. As the pipe diameter increases, the absolute roughness remains constant, resulting in a decreasing relative roughness (e/D). This leads to lower friction factors, which leads to larger velocities for a given pressure drop, and, thus, greater error.

We have also developed a more widely applicable tool than Figure 8 for assessing the error introduced by assuming incompressible flow. The more-appropriate parameter to relate gas-flow supply and discharge conditions is not the *L*/*D* ratio, but the ratio of *fL*/*D* (a choice commonly employed in gas-flow tabulations, and consistent with the arrangement of Equations [11] and [12]). Plotting the

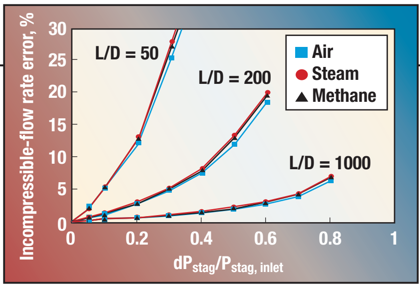


FIGURE 8. The pipe pressure-drop ratio and the ratio of pipe length to diameter are appropriate parameters for generalizing about the error that is introduced when assuming incompressible flow

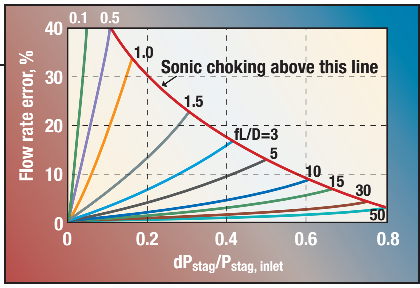


FIGURE 9. This map shows the error (overprediction) in flowrate prediction for a single pipe due to using incompressible-flow assumptions rather than an adiabatic compressible-flow calculation

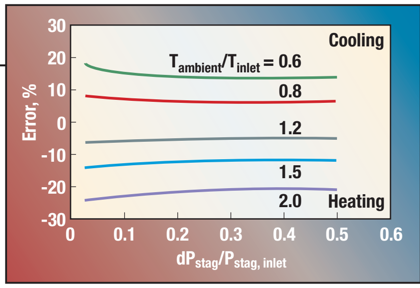


FIGURE 10. When a pipe is treated as adiabatic but actually has heat transfer, the flowrate prediction error can be sizable, even without an incompressible-flow assumption. The case here is for 100-psia, 70°F air entering an uninsulated steel pipe with L/D ratio of 200

incompressible-flow-assumption error against this parameter makes it possible to summarize the information on a single curve for each *fL/D* value, which applies for all pipe diameters.

Such an error map appears in Figure 9. It is based on an iterative program, Compressible Flow Estimator (CFE), developed by the author and being made available as a free download at http://www.aft.com/cfe.htm.

The results shown in Figure 9 are of general applicability. Various specific heat ratios, γ, and compressibility factors, Z, have been entered into the CFE, and the results always fall along the lines shown in Figure 9. This error map is also consistent with real-system predictions based on more-sophisticated calculation methods. Accordingly, Figure 9 is recommended to the engineer for general use as a guide in assessing compressibility in pipes.

Keep in mind, though, that Figure 9 assumes adiabatic flow. Additional error can result from flows involving heat transfer. The relative importance of heat transfer is addressed in the next section.

Finally, note that the direction of the incompressible-flow-assumption error is to overpredict the flowrate. Or, stated differently, for a given flowrate, it will underpredict the pressure drop. Unfortunately for typical pipe-system applications, neither of these conclu-sions is consistent with conservative design.

The sequence of steps that underlie the CFE program are available from the author. Also available from him are modified sequences, for handling situ-ations in which (1) the endpoint static pressure rather than the stagnation pressure are known, or (2) the temper-ature and flowrate are known but the endpoint stagnation pressure is not. ***Effect of heat transfer:*** The author knows of no general relationship showing the effect of heat transfer on the size of the incompressible-flow-assumption error. However, some insight can be gained from comparing relevant compressible-flow calculations (setting aside for a moment our preoccupation with the incompressible-flow-assumption error). Computer models were constructed to determine the difference in flowrate for air at different ambient temperatures.

The difference in flowrate for air with different ambient temperatures as compared to the compressible adiabatic case appears in Figure 10. It can be seen that cooling a gas may result in a greatly increased flowrate. In contrast, heating a gas can cause the flowrate to decrease significantly.

Accordingly, if an engineer is trying to design for a minimum flowrate, a gas stream that is cooling works in his or her favor by causing an underprediction of the flowrate when using adiabatic flow methods. When this error is combined with that of an incompressible-flow assumption, which overpredicts the flow, these two errors work in opposite directions, in part cancelling each other out. Conversely, a gas being heated adds further error on top of the incompressible-flow-assumption error, causing even more overprediction of the flowrate.

In many gas-pipe-system designs, the delivery temperature is as important as the delivery flowrate and pressure. In those cases, the heat-transfer characteristics of the pipe system take on the highest importance, and neither adiabatic nor isothermal methods —let alone incompressible-flow assumptions — can give accurate predictions. Unless the gas flow is very low and can be adequately calculated with incompressible methods, the designer is left with no choice but to perform a full compressible flow calculation. This means solving Equations (2)–(6) with a suitable relationship for the heat transfer to be used in Equation (4), or using more-convenient forms of these equations, such as Equations (13) or (14). Realistically, this requires appropriate software.

Network complications

When applying the concepts in this article, and in particular the use of the CFE program that underlies Figure 9, to a pipe network, the number of variables increases and the difficulty in assessing the potential error likewise increases. To investigate possible error-estimating methods, we have constructed simple flow models, one for incompressible flow and the other for compressible flow, of a manifolding pipe system. For simplicity, the compressible-flow model assumed that all flows are adiabatic. The basis is a 110-psia air system that enters a header and flows to three pipes at successive points along the header, terminating in a known pressure of 90 psia.

For each pipe in the system, the predicted *fL/D* and pressure-drop ratio have been determined from the incompressible-flow model. The resulting data have been entered into the CFE program for each pipe, and an approximate error generated for each. Then, starting from the supply, a path has been traced to each terminating boundary (of which there are three). The error for each pipe in the path has been summed, and then divided by the number of pipes in the path, giving an average error. This average has been compared to the actual difference between the results of the incompressible- and compressible-flow models.

Overall the comparison has proved favorable. However, applying CFE to this networked system underpredicts the actual error from the detailed models by up to 20%. The first pipe in the header shows the largest error, and the last pipe the smallest. As in the single-pipe calculations, the incompressible method overpredicts the flowrate.

In short, extra care should be taken when interpreting the meaning of incompressible-flow methods applied to gas pipe networks.

Rethinking the rules of thumb

The information presented up to now provides a basis for critiquing a number of rules of thumb upon which engineers often depend when dealing with gas flow.

Adiabatic and isothermal flow: One rule of thumb is the myth that adiabatic and isothermal flow bracket all flowrates. They do not, as has already been noted.

40%-pressure-drop rule: A common belief is what can be called the 40%-pressure-drop rule. Presented in a variety of handbooks, it states that if the pipe pressure drop in a compressible-flow system is less than 40% of the inlet pressure, then incompressible-flow calculation methods can be safely employed, with the average density along the pipe used in the equations.

In the handbooks, it is not made clear whether the pressure drop ratio is to be based on the stagnation or the static pressures. (In the author's experience, engineers apply the rule more frequently using stagnation-pressure ratios.) In any case, Figures 8 and 9 make it clear that the 40%-pressure-drop rule has no validity unless associated with a specific *L/D* ratio. Accordingly, this rule of thumb is highly misleading, and should be discarded by the engineering community.

Choked air flow at 50% pressure drop: An equation sometimes used as a rule of thumb to assess the likelihood of sonic choking is as follows (see, for instance, Reference [*4*], p 94):

$$\frac{p^*}{p_0} = \left(\frac{2}{\gamma+1}\right)^{\gamma/(\gamma-1)} \qquad \textbf{(15)}$$

where p^* is the critical static pressure at sonic velocity and p_0 the local stagnation pressure. For air, the specific heat ratio is 1.4, so the pressure ratio in the equation works out to 0.5283. This results in a pressure drop ratio of near 47% (in other words, about 50%) to bring about sonic choking. For gases with different specific heat ratios, the pressure drop ratio will differ somewhat, in accordance with Equation (15).

However, Equation (15) breaks down for pipe-system analysis when pipe friction becomes a factor. The reason is that the stagnation pressure in the equation is the pressure at the upstream side of the shock wave. If there is any pressure drop in the pipe from the supply pressure to the shock wave, then the supply pressure cannot be used in Equation (15). Instead, the local stagnation pressure at the shock wave must be used — but this is not known, unless the pressure drop is calculated using other means.

In short, Equation (15) cannot be used to predict the supply and discharge pressures necessary for sonic choking unless the piping has negligible friction loss.

Other simplified compressible-flow methods: A variety of simplified gas-flow equations, often based on assuming isothermal flow, crop up in the practical engineering literature. These typically have several drawbacks that are not always acknowledged or recognized:

- Most gas flows are not isothermal. In such cases, one cannot know how much error is introduced by the assumption of constant temperature. Related to this is the general issue of the importance of heat transfer on the gas flow, already mentioned
- Simplified equations typically do not address sonic-choking issues
- These equations are of no help when the delivery temperature is important
- The simplified equations break down at high Mach numbers
- Unrealistically, the entire pipe is solved in one lumped calculation, rather than using a marching solution
- It is difficult to extend the equations to pipe networks

In summary, simplified compressible-flow equations can be an improvement over assuming incompressible flow, but numerous drawbacks limit their usefulness.

Final thoughts

Compressors, blowers and fans raise the system pressure and density. These changes in properties inside the gas-flow system further limit the applicability of incompressible methods, beyond the cautions already discussed. Take special care in applying the incompressible-flow methods and estimation equations in this article to such systems.

The methods discussed in this article can help the engineer assess endpoint sonic choking, but restriction and expansion choking are somewhat more complicated. Accordingly, the estimation methods in this article may not be applied to all choking situations.

For new designs that require a lot of pipe, the engineer should consider the potential costs savings if smaller pipe sizes can be used. If significant cost savings prove to be possible, it may be prudent to invest in developing a detailed model that can more accurately determine the system capability over a range of pipe sizes. A detailed model may also help assess the wisdom of making modifications proposed for an existing system. ■

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